A METHOD FOR SIMULATING ELECTRICALLY MOVING TEMPERATURE FIELDS

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A simplified method, by comparison with that of [1], is given for simulating moving temperature fields using ohmic resistance electric network simulators.

The general form of the Kirchhoff–Fourier equation for nonstationary thermal conductivity in a moving medium is

$$\operatorname{div}(\lambda \operatorname{grad} T) - c\gamma\left(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}\right) + w = 0, \tag{1}$$

where v_X, v_Y, v_Z are velocity components of the motion of the medium at a given point.

The equation of the temperature field, referred to coordinates moving with velocity v_0 in the direction of the x-axis when λ, c, γ, $v_0 = -v_X$ are constants, is a particular case of (1) but is of sufficiently wide application in studying heat-transfer processes in welding and melting, continuous sequence heat treatment, mechanical material treatment, etc. With the above assumptions, Eq. (1) can be written

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} - \frac{1}{a}\frac{\partial T}{\partial t} + \frac{v_0}{a}\frac{\partial T}{\partial x} + \frac{w}{\lambda} = 0. \tag{2}$$

The initial and boundary conditions of the I–IV kind can be written in the same way as for a fixed coordinate system.

In [1] a method was given for simulating the solution of (2) electrically on ohmic resistance networks (R-networks), when the following substitution was used to deduce the parameters of the R-network:

$$T = u\varphi, \tag{3}$$

where

$$\varphi = \exp\left(-\frac{v_0 x}{2a}\right). \tag{4}$$

Then, from (2) we can obtain

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \frac{v^2}{4a^2}u - \frac{1}{a}\frac{\partial u}{\partial t} + \frac{w}{\lambda\varphi} = 0. \tag{5}$$

If we put

$$T = U\Phi, \tag{6}$$

where

$$\Phi = \exp\left(-\frac{v_0 x}{2a} - \frac{v_0^2 t}{4a}\right), \tag{7}$$

from (2) we can obtain the simpler equation

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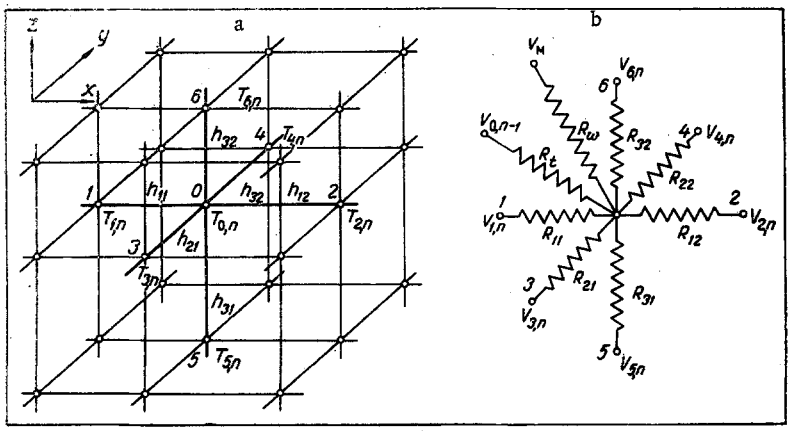

Fig. 1. Diagram of an elementary volume (a) and the nodes of an R-network for simulating the solution of (2) electrically.

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}-\frac{1}{a}\frac{\partial U}{\partial t}+\frac{w}{\lambda\Phi}=0, \tag{8}$$

i.e., there is no additional heat sink $((v_0^2/4a^2)u)$, which simplifies the construction and use of an electrical model. The boundary conditions of the I–IV kind correspondingly take the form

$$U_{s.n}=U_{me,n}, \tag{9}$$

$$\frac{\partial U_s}{\partial n}\Phi_s+\frac{\partial \Phi_s}{\partial n}U_s+\frac{q}{\lambda}=0, \tag{10}$$

$$\frac{\partial U_s}{\partial n}\Phi_s+\frac{\partial \Phi_s}{\partial n}U_s+\frac{\alpha}{\lambda}(U_{me}\Phi_{me}-U_s\Phi_s)=0, \tag{11}$$

$$\lambda\left(\frac{\partial U_s}{\partial n}\Phi_s+\frac{\partial \Phi_s}{\partial n}U_s\right)+\lambda_s\left(\frac{\partial U_{me}}{\partial n}\Phi_s+\frac{\partial \Phi_s}{\partial n}U_s\right)=0, \tag{12}$$

where the subscripts s and me refer to the surface and the medium. The remaining notation is in common use.

If we write (8)-(12) in finite difference form, as in [2], and compare with the expressions of Kirchhoff's law for the currents at the nodes of R-networks, we obtain an expression for calculating the parameters of the R-network which solves the system (8)-(12).

From the values of U which have been found, we can use (6) and (7) to determine T.

The determination of T can be simplified if the network parameters are computed so that we can obtain the values of T directly from the R-network.

We write Eq. (8) in finite difference form and for U we substitute its value from (6) (at the same time we multiply all the terms of the finite difference equations by $\prod_{i=1}^{3}\sum_{k=1}^{2}h_{ik}$ and cancel common factors)

$$\begin{aligned}
&\frac{2\left(T_{1,n}\dfrac{\Phi_{0,n}}{\Phi_{1,n}}-T_{0,n}\right)\prod\limits_{i=2,3}\sum\limits_{k=1}^{2}h_{ik}}{h_{11}\Phi_{0,n}}+\frac{2\left(T_{2,n}\dfrac{\Phi_{0,n}}{\Phi_{2,n}}-T_{0,n}\right)\prod\limits_{i=2,3}\sum\limits_{k=1}^{2}h_{ik}}{h_{12}\Phi_{0,n}}\\
&+\frac{2\,(T_{3,n}-T_{0,n})\prod\limits_{i=1,3}\sum\limits_{k=1}^{2}h_{ik}}{h_{21}\Phi_{0,n}}+\frac{2(T_{4,n}-T_{0,n})\prod\limits_{i=1,3}\sum\limits_{k=1}^{2}h_{ik}}{h_{22}\Phi_{0,n}}\\
&+\frac{2\,(T_{5,n}-T_{0,n})\prod\limits_{i=1,2}\sum\limits_{k=1}^{2}h_{ik}}{h_{31}\Phi_{0,n}}+\frac{2\,(T_{6,n}-T_{0,n})\prod\limits_{i=1,2}\sum\limits_{k=1}^{2}h_{ik}}{h_{32}\Phi_{0,n}}\\
&+\frac{\left(T_{0,n-1}\dfrac{\Phi_{0,n}}{\Phi_{0,n-1}}-T_{0,n}\right)\prod\limits_{i=1}^{3}\sum\limits_{k=1}^{2}h_{ik}}{a\delta t\Phi_{0,n}}+\frac{w\prod\limits_{i=1}^{3}\sum\limits_{k=1}^{2}h_{ik}}{\lambda\Phi_{0,n}}=0,
\end{aligned} \tag{13}$$

where the h_{ik} are space intervals on both sides of the node 0 (see Fig. 1); the axis x – i = 1; the axis y – i = 2; the axis z – i = 3. The time is $t = n\delta t$. In general the time interval δt may change during the solution.

The corresponding finite difference expressions for the boundary conditions of the II and III kind are:

$$\frac{T_s - T_m \frac{\Phi_{s,n}}{\Phi_{m,n}}}{h_m} + \frac{\partial \Phi_s}{\partial n_m} \frac{T_s}{\Phi_{s\,n}} + \frac{q}{\lambda} = 0, \tag{14}$$

$$\frac{T_s - T_m \frac{\Phi_{s,n}}{\Phi_{m,n}}}{h_m} + \frac{\partial \Phi_s}{\partial n_m} \frac{T_s}{\Phi_{s,n}} + \frac{\alpha}{\lambda}(T_m - T_s) = 0, \tag{15}$$

where h_m is the space interval along the normal to the surface (h_m is equal to that h_{ik} which approaches the node on the surface from the side of the body).

If there is to be similarity between (13)-(15) and the expressions for Kirchhoff's law for the currents flowing into the corresponding nodes of the R-network, it is necessary that the parameters of the R-network should be as follows:

$$R_{11} = \frac{h_{11}\Phi_{0,n}}{2\prod_{i=2,3}\sum_{k=1}^{2} h_{ik}} R_N, \quad R_{32} = \frac{h_{32}\Phi_{0,n}}{2\prod_{i=1,2}\sum_{k=1}^{2} h_{ik}} R_N,$$

$$R_t = \frac{a\delta t \Phi_{0,n}}{\prod_{i=1}^{3}\sum_{k=1}^{2} h_{ik}} R_N, \quad R_w = \frac{(V_M - V_{0,n}) K\lambda\Phi_{0,n}}{w\prod_{i=1}^{3}\sum_{k=1}^{2} h_{ik}} R_N, \tag{16}$$

$$R'_q = \frac{(V_M - V_{0,n}) K\lambda R_{ik}}{h_{ik} q}, \quad R''_q = \frac{\Phi_s R_{ik}}{\frac{\partial \Phi_s}{\partial n_m} h_{ik}};$$

$$R'_\alpha = \frac{\lambda R_{ik}}{\alpha h_{ik}}, \quad R''_\alpha = \frac{\Phi_s R_{ik}}{\frac{\partial \Phi_s}{\partial n_m} h_{ik}},$$

where the resistances $R_{11}, \ldots, R_{32}$ correspond to the space intervals $h_{11}, \ldots, h_{32}$ (see Fig. 1) and simulate the thermal heat conduction resistance; R_t is the resistance through which passes the current simulating heat absorption or emission by the due to the heat capacity of an elementary volume $\prod_{i=1}^{3}\sum_{k=1}^{2} h_{ik}$ in a time interval δt. One end of the resistance R_t is attached to the node, the other to a voltage divider from which the voltage $V_{0,n-1}$ is taken.

At the second end of the resistance R_w, voltage V_M is applied to R'_q. If $V_M \gg V_{0,n}$, when w, q are heat sources, or $V_M \ll V_{0,n}$, when w, q are heat sinks, instead of $(V_M - V_{0,n})$ in the expressions for R_w, R_q we may put V_M. K is the scale of the transition from temperature to voltage:

$$K = \frac{T_{max} - T_{min}}{V_{max} - V_{min}}.$$

For given boundary conditions of the I kind, voltages are applied to the nodes corresponding to T_s. At the free end of the resistance R'_α a voltage is applied corresponding to the temperature T_{me} of the medium. At the free ends of the resistances R''_q and R''_α the minimal voltage $V_M = 0$ is applied. If $T_{me} = 0$ then

$$R_\alpha = \left[\frac{1}{R'_\alpha} + \frac{1}{R''_\alpha}\right] = R_{ik}\left[h_m\left(\frac{\alpha}{\lambda} + \frac{1}{\Phi_s}\frac{\partial \Phi_s}{\partial n_m}\right)\right]^{-1}.$$

For given boundary conditions of the IV kind, the networks simulating the bodies in contact are joined to each other.

If $\partial\Phi_s/\partial n_m = 0$, which occurs if the normal to the surface lies in the plane perpendicular to the x-axis, the boundary conditions of the II and III kind are simulated using only the resistances R'_α and R'_q.

The accuracy of the solution depends on the ratios Φ_0/Φ_1 and Φ_0/Φ_2; the closer they are to unity, the higher is the accuracy. The ratios Φ_0/Φ_1 and $\Phi_0/\Phi_2 \to 1$ for h_{1k} = const and $x \to \infty$ or $h_{1k} \to 0$ and x = const, i.e., for network nodes with small values of x we have to choose appropriately small values of h_{1k} and for nodes with large values of x, the intervals h_1 and h_2 can be increased.

A similar discussion holds for $\Phi_{0,n-1}/\Phi_{0,n}$.

In solving problems with moving concentrated heat sources the origin is linked to the position of the sources.

The quasistationary situation, when $\partial T/\partial t = 0$ was discussed in [1].

In a similar manner the parameters of the R-network can be deduced for the original set of equations in nondimensional form.

If it cannot be assumed that λ, c, γ, v are constant, the solution of Eq. (1) in the most general formulation can be obtained by complicating the technique of solving on an R-network or by a combined model using the general method of [2], or on a special quasianalog model developed in [3, 4].

LITERATURE CITED

1. L.A.Kozdoba and V.I.Makhnenko, Inzh.-Fiz. Zh., 4, No.11 (1961).
2. L.A.Kozdoba, The Electrical Simulation of Temperature Fields for Components in Marine Power Plants [in Russian], Sudostroenie, Leningrad (1964).
3. A.V.Temnikov, B.M.Gavrilov, and V.V.Basov, Izv. Vuzov, Priborostroenie, No.5 (1966).
4. A.N.Reznikov, A.V.Temnikov, N.V.Diligenskii, and B.M.Gavrilov, in: Mathematical Simulation and the Theory of Electrical Circuits [in Russian], No.5, Naukova Dumka, Kiev (1967).